

PROBLEM-SOLVING

Introduction

We have looked at the major investigative processes and how to get the most common kinds of investigations working in the classroom.

In this chapter we shall look at some other important problem-solving processes. These are more broadly defined than the investigative processes.

What is problem-solving?

Problem-solving is, expressed very simply, what one does when one has a problem, and wants to solve it. Problem-solving is directed towards a goal. The statement of the problem indicates what type of goal is being aimed for.

e.g.

Problem: Design a timetable for your class.

Goal: A completed and workable timetable.

Problem: Write a magazine article to explain your favourite sport or game.

Goal: An article which would make sense to someone who originally knew nothing about the sport or game concerned.

Although there is a goal in problem-solving, there is more than one possible answer. You could imagine more than one possible class timetable and more than one magazine article.

Problem-solving, in the sense used in the National Curriculum, is applied problem-solving. Both the starting question and the goal have relevance outside mathematics. Mathematics is used as a tool on the journey from problem statement to goal. We shall also mention briefly 'pure' problem-solving where the goal lies within mathematics and may not have any immediate relevance to a real-world problem.

A certain kind of maths problem has been around for a long while. The problems are usually artificial, designed to give students the opportunity to practise a particular technique. Learning how to do them will not help students cope with maths in the real world.

In stereotypical problems, all the data needed for solving is given in the statement of the problem, and the data given in the problem is *all* needed to solve it. There is an intended, particular, technique for solving the problem and there is only one correct answer.

Problems outside the mathematics classroom—real problems—rarely come with all the information needed to solve them. It is for the problem-solver to decide what data is necessary to help solve them, and to collect, process and manipulate that data to find a solution.

Data, in the sense used here, could, for instance be scientific measurements. They could be data in the normal sense of the word, i.e. collected by surveys or other kinds of research, or your own thoughts about the problem.

Problems outside the mathematics classroom do not indicate which parts of mathematics would help with their solution. Therefore the teaching of skills in isolation and their practice in solving specially designed problems is insufficient. Students need to learn to identify the mathematical aspects of a problem and to apply their mathematical skills.

The more decisions students make for themselves when they are solving problems, the more the problems mirror those which they will face outside the mathematics classroom. Although the teacher must provide some guidance, it is not their instruction but the students' own decision-making and involvement with the problem which continuously generates the work. The students work towards a particular goal, usually in an applied context. The goal can be set by the teacher, by teaching materials, or identified by the students themselves. Once this is done, students decide how to pursue the task. They then work towards a solution (or solutions), recording and presenting their results. The method of presentation can be prescribed by the teacher or by materials, or students can present their results in their own way.

This open type of problem-solving provides:

- an opportunity to learn and use aspects of mathematics such as identifying relevant mathematical information and selecting appropriate techniques;
- meaningful contexts for the application of mathematics;
- an opportunity to fulfil cross-curricular aims related to decision-making.

Problem-solving provides good motivation for the learning of mathematics. When students are solving problems, and they identify the need for a particular skill or item of knowledge, they are well motivated to learn it. Such learning is more likely to become part of the students' mathematical repertoire than that which takes place outside any relevant context. This learning is thus more secure, and mathematics learnt in this way can be applied to other relevant situations.

Opening Up Problems

A useful way to begin to think about more open problems is to start with conventional closed problems. These can be made more open by removing some information. This means that a problem now has more than one solution. The student may invent or research the missing information, or give different answers for a range of different values of the missing information.

Example

How long would it take a car travelling at an average speed of 55 m.p.h. to cover 185 miles?

This could be made more open:

How long would you expect it to take to get from Manchester to London by car?

This is similar to the examples shown on page D7 of the *Non-Statutory Guidance*. Those examples, however, are not in applied contexts.

✱(✱) Exercise

Open up the following problems in a similar way:

- 1) A woman invests £2500 in a high interest building society account. She can expect a return of 8.5% per annum. She opts to have the interest kept in the account. How much money would there be in the account after 10 years?
- 2) A room measures 12' 9" by 16' 4" into the alcoves. There are two alcoves. They are in one of the shorter walls, at either end of the wall. They measure 3' 6" wide by 1' deep. Make a scale drawing of the room using a scale of 1" to represent 1'.
- 3) A horse called St. Brendan is running in the 2.30. His odds are 12:5. Another horse called Florrie is running in the 3 o'clock. Her odds are 3:1. What is the probability that both horses will win their races?

continued

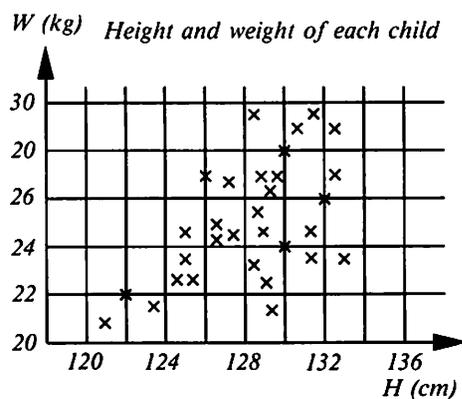
Exercise continued

- 4) The following table shows the marks Parminder got in each of her exams:

Subject	Mark	Subject	Mark
Mathematics	90%	English	89%
Geography	45%	History	67%
Science	87%	Des/Tech	73%
Urdu	94%	French	88%
Social Studies	56%		

Draw a histogram to show Parminder's marks in her exams.

- 5) Draw a line of best fit on this scattergram:



Reproduced from *Mathematics in The National Curriculum p.40*

Some Possible Responses to this Exercise

- 1) Which of your high street building societies would give the best return on £2500 after 10 years?
- 2) Make a scale drawing of the classroom or of a room at home.
- 3) Look at the odds of the horses in a particular race meeting. You have been given £5 to gamble, on condition that you 'do an accumulator'. That means you bet on one horse and if you win, your winnings are automatically bet on a horse of your choice in a subsequent race. Choose your two horses for your accumulator. Explain your choice. What is the probability of winning? How much would you win?
- 4) Draw a graph to show your exam results last year.
- 5) If you know someone's weight, can you say anything about how tall they are likely to be?

Problem-Solving Stages

It is possible to identify three major stages in the problem-solving procedure. At least one process from each stage will be used whether the problem is one with a prescribed method of presentation, set by the teacher, a very open problem devised by the students themselves, or anything between the two.

- Stage 1*
- Problem-posing (i.e. choosing what question(s) to pursue)
 - Refining or clarifying a question
 - Making sense of a given task
 - Identifying the mathematical aspects of a problem
 - Planning how to pursue the problem
 - Choosing appropriate mathematical techniques
- Stage 2*
- Collecting data
 - Making sense of given data
 - Calculating
 - Measuring
 - Manipulating data
- Stage 3*
- Presenting data
 - Finding or creating a solution
 - Presenting a solution
 - Interpreting a solution
 - Evaluating a solution (possibly by simulating its use)
 - Putting a solution into practice
 - Using results or a solution for some other purpose

Stage 3 can lead to the posing of new questions, making the whole procedure cyclical. The stages are not necessarily separate, and students can sometimes be seen amending questions and altering the task in hand as they proceed. This is particularly the case in extended problem-solving.

Some problem-solving activities can go on for several weeks. Shorter problems can be completed within one lesson—these tend to be tasks where the teacher sets the question. The choice of how to pursue the task, and often how to present the solution, can still rest with the students.

* Exercise

Use one of the following problems with a group of students:

- *Lift Regulations* (p.92)
- *Sports Rules* (p.105)
- *Washington Underground* (p.114)

Explain to the class at the beginning of the lesson that this is rather like the investigations they have been doing; they will have to decide how to go about it themselves. But in this case they are expected to produce an answer to the problem. There is no one right answer. They must produce an answer which they are happy with.

Problem-Posing

There are problems which, without asking specific questions, invite students to research within a particular area of study, e.g.:

'Do a vehicle survey.'

Such tasks can be frustrating and seem to be a waste of time unless the students are clear about *why* they are gathering data or investigating. Often the hidden question is:

'Can you use what you find out to make predictions?' Meteorologists and other scientists often work in this way. When this is not the hidden question the students should be advised to define a specific question for themselves. For example, when asked to do the vehicle survey they could ask:

'Do teachers have older cars?'

or *'Are there more lorries than cars going down the road outside our school?'*, etc.

Suppose a task required them to survey a cemetery. They could ask:

'Do women live longer than men?', etc.

It is not fair to let students waste time gathering data for the sake of it. They should always have a clear purpose. Another example of a problem without a question is provided by part of an examination that was set for a Mode 3 CSE in 1967, and shows how long these ideas have been around. The examination started at 9.30, had no time limit, and the candidates were asked to choose one question from five. The first question was:

Write about the mathematics of a chessboard (or draught board).

The first thing students would have to do in attempting this, would be to define the question they chose to pursue. The question that most mathematics teachers would probably expect would be:

* *How many squares are there?*

but there are also many other possible questions, for example:

* *How many rectangles are there?*

* *Can a knight visit each square once and return to its starting point?*

* *Can it be covered with 31 dominoes if two squares are removed?*

Not all of these are equally fruitful in terms of the mathematics they generate, but that is not the point. The point is that a student attempting this activity is given experience of defining problems, as well as solving problems that are posed by someone else.

Pure Problems and Real-World Problems

The chessboard example mentioned previously is a ‘pure’ problem. The original stimulus came from the outside world but the spirit in which the inquiry is conducted is quite remote from the real world. It doesn’t really matter whether the number of squares on a chessboard is 200 or 204. What is important is the process of finding out. This contrasts with ‘real-world’ problems, where the concern is getting an answer which we can be reasonably certain of.

This distinction is nicely illustrated by the following problem (which occurred in September 1989):

- *A letter was weighed for posting*
- *It was found that it would cost 97p to send first class*
- *In the office only 14p and 19p stamps were available*
- *Can 97p be made with 14p and 19p stamps?*

The following table shows that it can’t be done exactly:

		number of 14p stamps							
		0	1	2	3	4	5	6	7
number of 19p stamps	0	0	14	28	42	56	70	84	98
	1	19	33	47	61	75	89		
	2	38	52	66	80	94			
	3	57	71	85	99				
	4	76	90						
	5	95							

At this point, the mathematical activity splits into (at least) two sorts of activity. The ‘pure’ questions at this point might be:

- What amounts cannot be made?*
- What is the largest amount that cannot be made?*
- How many amounts cannot be made?*

In all these cases, the reference is towards the world of mathematics.

Problem continued

In contrast to this the 'real' or 'applied' question would be:

What is the smallest wastage?

A quick glance at the table shows that 98p can be made exactly, using only 14p and 19p stamps, so the practical solution would be to stick seven 14p stamps on the envelope and post it. At this point another question may arise:

What is the smallest wastage for any amount of money?

It is important to realise, however, that this is another 'pure' question. At these rates of postage, there is simply no need to know how to make up, for example, 70p, since it is not in the table of rates for letters. If, on the other hand, you were working in an office, and only had access to 14p and 19p stamps, and were constantly having to make these sorts of calculations in a hurry to catch the last post, then it might be worthwhile making up a table for quick reference.

Here are the postal rates for letters up to 30th September 1989.

Weight not over	First class	Second class
60	19	14
100	28	22
150	34	26
200	42	32
250	51	39
300	59	46
350	67	52
400	76	59
450	86	65
500	97	73
600	120	90
700	140	105
800	160	
900	175	not available
1000	190	over 750g
Each extra 250g or part thereof	45	

From October 1st, 1989, the first and second class postage rates became 20p and 15p respectively. For the first time for many years the first and second class letter rates have a factor in common. What this means for this particular problem depends on whether your view is pure or applied, the pure problem becomes trivial, while the sensible approach in the real world is to stock up with 15p and 5p stamps, which means that the maximum wastage will be never more than 4p.

Evaluation

When students are solving applied problems it is important that they evaluate their solutions. This is a normal part of real problem-solving outside the mathematics classroom. The best kind of evaluation of a solution is to answer the question:

'What would happen if this solution were put into practice?'

In some situations it has been possible to try this out, e.g. at Cabrillo College in California the students designed the plan for the new car-park, a group of primary school children in the UK solved a production line problem for Parker (Pens) International and in one school the children designed the school menu.

The question: *'What would happen if we put this solution into practice?'* will not work for all types of problems. However, it is important that students do not regard finding a solution as finishing the problem. They should always be encouraged to look at their solution and make sure that it is their best and that they cannot improve on it.

Sources for Problem-Solving

There are few published materials specifically designed for the kind of problem-solving we have been dealing with. But any resources can be made into problem-solving resources. Getting students started with questions to pursue is the beginning. Data-collecting tasks are a good way to start. Questions to pursue and data to gather could come from other subjects, or students could devise them in mathematics lessons.

Students do come up with questions in the course of lessons. Watch out for these and capitalise on them. Students could then pursue problems which they are curious about. This will increase their motivation.

As this process continues you may find that the mathematics going on in your classroom becomes less dependent on published tasks and based more on real problem-solving.

'Activities should where appropriate, use pupils' own interests or questions, either as starting points or as further lines of development.'

There is abundant evidence that work which is triggered by pupils themselves leads to better motivation and understanding. Pupils' remarks and suggestions, based on, say, activities from outside the classroom, and local and national news, all have their place in bringing mathematics to life and making it relevant. Teachers should have sufficient flexibility built into their schemes to capitalise on their pupils' interests.'

page B9, Non-Statutory Guidance